SOLUTIONS:

Exercise 1

The theorem of Cauchy gives the expression that links the stress vector, stress tensor and normal vector on the surface passing by the point P $\boldsymbol{n} = (n_1, n_2, n_3)^T$:

$$t_i = \sigma_{ij}\, n_j$$

or

$$t = \begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix} = \begin{pmatrix} \sigma_{11} & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} \sigma_{11} n_1 + 2n_2 + n_3 \\ 2n_1 + 2n_3 \\ n_1 + 2n_2 \end{pmatrix}$$

A plane free of stress exists if the normal vector to it $\boldsymbol{n}^* = (n_1^*, n_2^*, n_3^*)^T$ satisfies

$$\begin{cases} \sigma_{11} n_1^* + 2n_2^* + n_3^* = 0 \\ 2n_1^* + 2n_3^* = 0 \\ n_1^* + 2n_2^* = 0 \end{cases}$$

That implies,

$$n_3^* = -n_1^* \quad , \quad n_2^* = -\frac{1}{2} n_1^* \quad , \quad (\sigma_{11} - 2) n_1^* = 0$$

Using also the condition, $(n_1^*)^2 + (n_2^*)^2 + (n_3^*)^2 = 1$, we obtain, $(n_1^*)^2 \left(1 + \frac{1}{4} + 1\right) = 1$

Or,

$$n^* = \pm \frac{1}{3} (2, -1, -2)^T \quad , \quad \sigma_{11} = 2$$

Exercise 2

For the considered plate element, the equations of equilibrium are

$$\begin{aligned}0 &= \sigma_{11}\left(x_1+dx_1,x_2\right)\,dx_2-\sigma_{11}\left(x_1,x_2\right)\,dx_2\\&+\sigma_{12}\left(x_{1,}x_2+dx_2\right)\,dx_1-\sigma_{12}\left(x_{1,}x_2\right)\,dx_1\end{aligned} \quad ,$$

$$\begin{aligned}0 &= \sigma_{22}\left(x_1,x_2+dx_2\right)\,dx_1-\sigma_{22}\left(x_1,x_2\right)\,dx_1\\&+\sigma_{21}\left(x_1+dx_1,x_2\right)\,dx_2-\sigma_{21}\left(x_{1,}x_2\right)\,dx_2\end{aligned}$$

Dividing the last expression by dx_1dx_2, we obtain

$$\frac{\sigma_{11}\left(x_1+dx_1,x_2\right)\;-\sigma_{11}\left(x_1,x_2\right)}{dx_1}+\frac{\sigma_{12}\left(x_{1,}x_2+dx_2\right)\;-\sigma_{12}\left(x_{1,}x_2\right)}{dx_2}=0 \quad ,$$

$$\frac{\sigma_{22}\left(x_{1,}x_2+dx_2\right)\;-\sigma_{22}\left(x_{1,}x_2\right)}{dx_2}+\frac{\sigma_{21}\left(x_1+dx_1,x_2\right)\;-\sigma_{21}\left(x_1,x_2\right)}{dx_1}=0$$

When $dx_1\rightarrow 0$ et $dx_2\rightarrow 0$, we have

$$\frac{\partial\sigma_{11}}{\partial x_1}+\frac{\partial\sigma_{12}}{\partial x_2}=0 \quad , \qquad \frac{\partial\sigma_{21}}{\partial x_1}+\frac{\partial\sigma_{22}}{\partial x_2}=0$$

The equilibrium of the moments around the center of the elements results in the following equation

$$\begin{aligned}0 &= \frac{1}{2}\sigma_{21}\left(x_1+dx_1,x_2\right)\,dx_2dx_1+\frac{1}{2}\sigma_{21}\left(x_1,x_2\right)\,dx_2dx_1\\&-\frac{1}{2}\sigma_{12}\left(x_{1,}x_2+dx_2\right)\,dx_2dx_1-\frac{1}{2}\sigma_{12}\left(x_{1,}x_2\right)\,dx_2dx_1\end{aligned}$$

Dividing this last equation by dx_1dx_2 and considering $dx_1\rightarrow 0$ et $dx_2\rightarrow 0$, we have

$$\sigma_{21}=\sigma_{12}$$

Exercise 3

Using the notation, $f_{i,j} = \frac{\partial f_i}{\partial x_j}$, the equilibrium equations can be written as

$$\sigma_{ij,j} + \rho b_i = 0$$

To check if volume forces act on the solid, we need to verify that $\sigma_{ij,j} = 0$

To advance, we notice that $\frac{\partial R}{\partial x_j} = R_{,j} = \frac{x_j}{R}$

With the given stresses, σ_{ij}, we have

$$\sigma_{ij,j} = \left[\frac{\alpha x_i x_j x_3}{R^5}\right]_{,j}$$

$$= \frac{1}{R^5}\left(\alpha x_i x_j x_3\right)_{,j} + \alpha x_i x_j x_3 \left(\frac{1}{R^5}\right)_{,j}$$

$$= \frac{\alpha}{R^5}\left(x_{i,j} x_j x_3 + x_i x_{j,j} x_3 + x_i x_j x_{3,j}\right) - 5\alpha x_i x_j x_3 \frac{1}{R^6}\frac{x_j}{R}$$

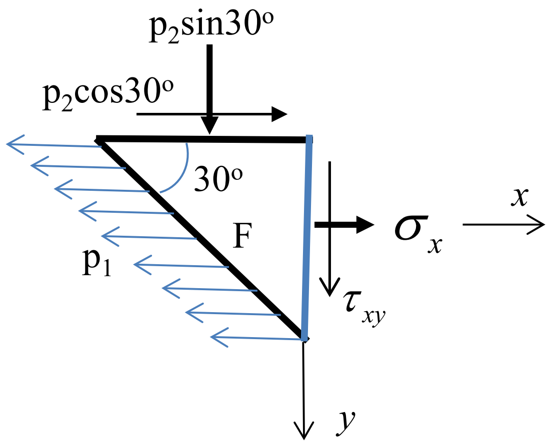
Also noticing that $x_{i,j} = \delta_{ij}$, $x_{3,j} = \delta_{3j}$ et $x_j x_j = R^2$, we obtain

$$\sigma_{ij,j} = \frac{\alpha}{R^5}\left(x_i x_3 + 3x_i x_3 + x_i x_3 - 5x_i x_3\right) = 0$$

Thus, the equilibrium is satisfied without body forces.

Exercise 4

(a): Recall that stress analysis is carried out on two normal planes through the point of interest. In this particular geometry, we consider a vertical cut from the lower left corner,



Equlibrium along x

$$\sigma_x F \sin 30^o - 150F + (70 \cos 30^o) F \cos 30^o = 0$$

$$\Rightarrow \sigma_x = \frac{150 - (70 \cos 30^o) \cos 30^o}{\sin 30^o} = 195 MPa$$

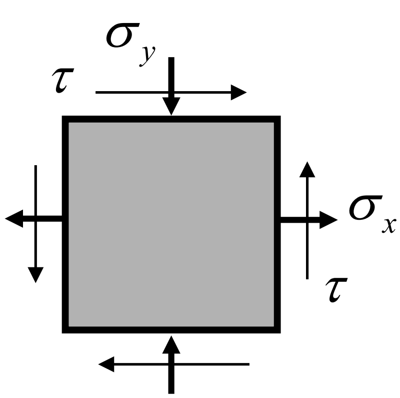
Equlibrium along y

$$\tau_{xy} F \sin 30^o + (70 \sin 30^o) F \cos 30^o = 0$$

$$\Rightarrow \tau_{xy} = -70 \cos 30^o = -60.62 MPa$$

$\Rightarrow \tau = 60.62 MPa; \quad \sigma_x = 195 MPa; \quad \sigma_y = 35 MPa$ (Negative as indicated)

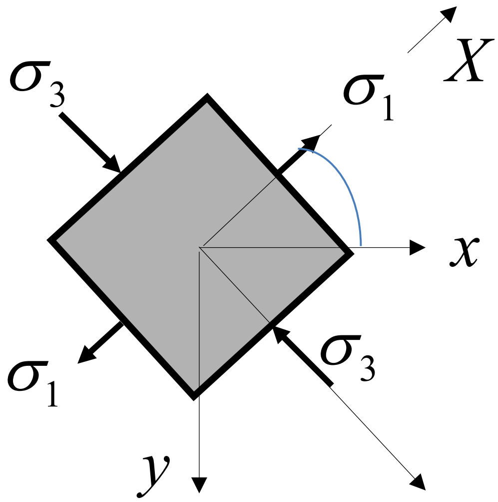
(b): Principal stresses



$$\sigma_{1,2} = \frac{195-35}{2} \pm \sqrt{\left(\frac{195+35}{2}\right)^2 + 60.62^2} = 80 \pm 130 MPa$$

$$\Rightarrow \sigma_1 = 210 MPa; \quad \sigma_2 = -50 MPa;$$

Their orientation is



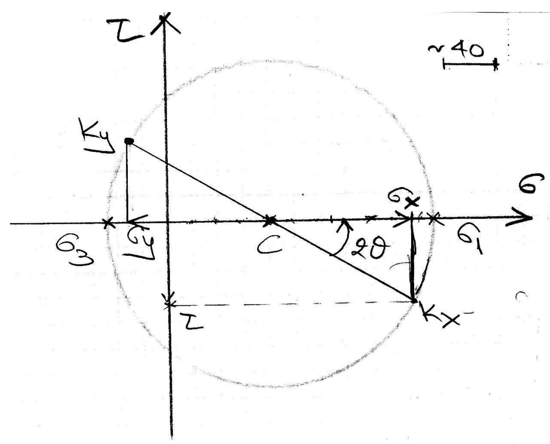
$$\tan 2\theta = \frac{2\tau}{\sigma_x - \sigma_y} = \frac{2(60.62)}{195+35} = 0.526 \Rightarrow \theta = 13.89^o$$

(c)

For the Mohr's cycle (units in MPA):

Center $C\left(\frac{\sigma_x+\sigma_y}{2},0\right) = C(80,0)$

Radius $R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau^2} = 130$



Note here that the shear stress on the positive x-face is negative (opposite to the positive direction). From the circle, we can deduce the same values calculated with the expressions.

Exercise 5

The direction of the octahedral plane is defined with its unit normal vector in the principal stress space (i.e., the plane has equal direction cosines with respect to the three principal directions, $\boldsymbol{n}_1, \boldsymbol{n}_2, \boldsymbol{n}_3$),

$$\boldsymbol{m} = \frac{1}{\sqrt{3}}\boldsymbol{n}_1 + \frac{1}{\sqrt{3}}\boldsymbol{n}_2 + \frac{1}{\sqrt{3}}\boldsymbol{n}_3$$

Thus, the stresses on that plane are given by the Cauchy's expression,

$$\begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix}$$

$$t_N = m_i t_i = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) \frac{1}{\sqrt{3}} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{I_1(\sigma)}{3}$$

$$t_T = \left(t_i t_i - t_N^2\right)^{1/2} = \left[\left(\frac{\sigma_1^{\ 2} + \sigma_2^{\ 2} + \sigma_3^{\ 2}}{3}\right) - \left(\frac{I_1(\sigma)}{3}\right)^2\right] = \frac{1}{3}\sqrt{3\left(\sigma_1^{\ 2} + \sigma_2^{\ 2} + \sigma_3^{\ 2}\right) - I_1^2(\sigma)}$$

$$= \frac{1}{3}\sqrt{3\left(\sigma_1 + \sigma_2 + \sigma_3\right)^2 - 6\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right) - I_1^2(\sigma)} = \frac{1}{3}\sqrt{2I_1^2(\sigma) - 6I_2(\sigma)}$$

Alternatively, we can replace the first invariant to obtain a well-known expression for the octahedral stress,

$$t_T = \frac{1}{3}\sqrt{3\left(\sigma_1^{\ 2} + \sigma_2^{\ 2} + \sigma_3^{\ 2}\right) - I_1^2(\sigma)}$$

$$= \frac{1}{3}\sqrt{3\left(\sigma_1^{\ 2} + \sigma_2^{\ 2} + \sigma_3^{\ 2}\right) - \left(\sigma_1^{\ 2} + \sigma_2^{\ 2} + \sigma_3^{\ 2} + 2\sigma_1\sigma_2 + 2\sigma_2\sigma_3 + 2\sigma_3\sigma_1\right)}$$

$$= \frac{1}{3}\sqrt{2\left(\sigma_1^{\ 2} + \sigma_2^{\ 2} + \sigma_3^{\ 2}\right) - \left(2\sigma_1\sigma_2 + 2\sigma_2\sigma_3 + 2\sigma_3\sigma_1\right)}$$

$$\Rightarrow t_{oct} = t_T = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

Exercise 6

1. The unit normal vector to the plane intersecting the point P is

$$\boldsymbol{n} = \frac{\nabla f(x_p)}{\left\|\nabla f(x_p)\right\|} = \frac{2e_1 + e_2 - 3e_3}{\sqrt{14}}$$

The Cauchy relation gives the stress vector (B3.76)

$t_i = \sigma_{ij} n_j$, i.e.

$$\begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix} = \frac{1}{\sqrt{14}} \begin{pmatrix} 20 & 10 & -10 \\ 10 & 30 & 0 \\ -10 & 0 & 50 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} = \frac{1}{\sqrt{14}} \begin{pmatrix} 80 \\ 50 \\ -170 \end{pmatrix}$$

From (B3.108) we have

$$t_N = n_i t_i = \frac{1}{\sqrt{14}} \frac{1}{\sqrt{14}} \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}^T \begin{pmatrix} 80 \\ 50 \\ -170 \end{pmatrix} = \frac{1}{14}(160 + 50 + 510) = 51.42\,\text{MPa}$$

$$t_T = \left(t_i t_i - t_N^2\right)^{1/2} = \left[\frac{1}{14}\left(80^2 + 50^2 + 170^2\right) - \left(51.42\right)^2\right]^{1/2} = 7.41\,\text{MPa}$$

2. Using the expressions are (B3.115-117) and the given values of stresses we obtain

$$I_1(\sigma) = 20 + 30 + 50 = 100$$

$$I_2(\sigma) = \begin{vmatrix} 20 & 10 \\ 10 & 30 \end{vmatrix} + \begin{vmatrix} 30 & 0 \\ 0 & 50 \end{vmatrix} + \begin{vmatrix} 20 & -10 \\ -10 & 50 \end{vmatrix} = 2{,}900$$

$$I_2(\sigma) = \begin{vmatrix} 20 & 10 & -10 \\ 10 & 30 & 0 \\ -10 & 0 & 50 \end{vmatrix} = 22{,}000$$

From (B3.114) we have for the characteristic equation

$$\lambda^3 - 100\lambda^2 + 2{,}900\lambda - 22{,}000 = 0$$

Exercise 7.1

1. One of the principal stresses is $\sigma_3 = -50$ MPa because on plane 3, the shear stresses are zero ($\sigma_{13} = \sigma_{23} = 0$). As for the other two, we use the equations applied for a two-dimensional problem,

$$\sigma_{1,2} = \frac{\sigma_{11} + \sigma_{22}}{2} \pm \sqrt{\left(\frac{\sigma_{11} - \sigma_{22}}{2}\right)^2 + \sigma_{12}^2} \quad \Rightarrow \quad \sigma_1 = 108.28MPa; \quad \sigma_2 = 51.70MPa$$

$$\tan 2\theta = \frac{2\sigma_{12}}{\sigma_{11} - \sigma_{22}} \quad \Rightarrow \quad \theta = 22.5^o$$

2. Maximum shear

With the three principal stresses known we get

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} \quad \Rightarrow \quad \frac{108.3 - (-50)}{2} = 79.15MPA$$

3. The octahedral stress

Since we have the principal stress-values we can use,

$$t_{oct} = t_T = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

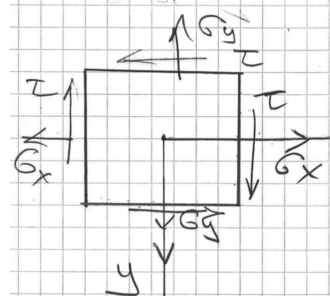
Thus, $t_{oct} = t_T = \frac{1}{3}\sqrt{(108.28 - 51.70)^2 + (51.70 + 50)^2 + (-50 - 108.28)^2} = 65.48MPa$

Exercise 7.2

Series 3

Exercise 7.2

The stress state is



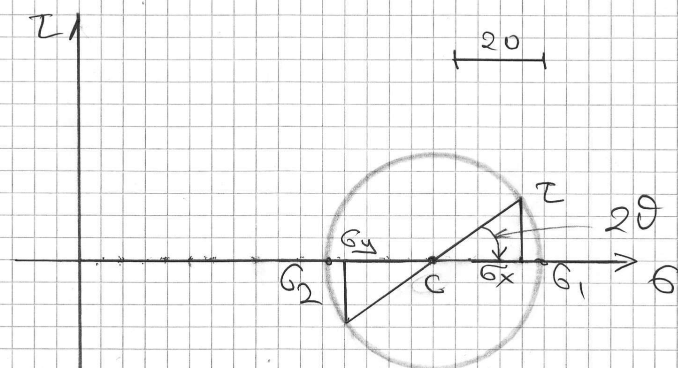
$$\begin{pmatrix} \sigma_x & \tau \\ \tau & \sigma_y \end{pmatrix} = \begin{pmatrix} 100 & 20 \\ 20 & 60 \end{pmatrix}$$

$\sigma_x \rightarrow \sigma_{11}$

$\sigma_y \rightarrow \sigma_{22}$, $\tau \rightarrow \sigma_{12}$

Mohr's Cycle parameters (see solution of Exo 4)

$C(80,0)$; $R = 28.30$



$\Rightarrow$ $\sigma_1 \cong 108$ MPa

$\sigma_2 \cong 52.$ MPa

$$\tan 2\theta = \frac{2\tau}{\sigma_x - \sigma_y}$$

Note that τ is positive here (i.e. clockwise from x to y is positive)

Exercise 8

From the given data we have, σ_1 = 56; σ_2=35; σ_3=14. Thus,

1. $\tau_{\max} = \dfrac{\sigma_1 - \sigma_3}{2}$=21; its orientation is along the angle bisector (i.e. 45°) of axes 1 and 3.

2. We use the expression, $t_{oct} = t_T = \dfrac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$ and the given principal stresses to obtain $t_{oct} = 17.15$.

Exercise 9

From (B3.114) we have for the characteristic equation,

$$\begin{vmatrix} 3-\lambda & 1 & 1 \\ 1 & -\lambda & 2 \\ 1 & 2 & -\lambda \end{vmatrix} = \lambda^3 - 3\lambda^2 - 6\lambda + 8 = 0$$

The last equation can be expressed as ,

$$\lambda^3 - 3\lambda^2 - 6\lambda + 8 = (\lambda+2)(\lambda-4)(\lambda-1) = 0$$

From where we obtain the principal values,

$$\sigma_1 = 4; \quad \sigma_2 = 1; \quad \sigma_3 = -2$$

The principal direction for the value $\sigma_3 = -2$ is calculated as follows (see example 1.8 in Botsis and Deville),

$$(3+2)(n_3)_1 + (n_3)_2 + (n_3)_3 = 0$$
$$(n_3)_1 + 2(n_3)_2 + 2(n_3)_3 = 0$$
$$(n_3)_1 + 2(n_3)_2 + 2(n_3)_3 = 0$$

From the first two equations, we have $(n_3)_1 = 0; \quad (n_3)_2 = -(n_3)_3$. Using this result in the orthogonality condition,

$$(n_3)_1^2 + (n_3)_2^2 + (n_3)_3^2 = 1$$

we obtain, $(n_3)_1 = 0; \;\; (n_3)_2 = 1/\sqrt{2}; \;\; (n_3)_3 = -1/\sqrt{2}$

Similarly, for the other principal values we have,

$\sigma_2 = 1$; $(n_2)_1 = 1/\sqrt{3}$; $(n_2)_2 = -1/\sqrt{3}$; $(n_2)_3 = -1/\sqrt{3}$

$\sigma_1 = 4$; $(n_1)_1 = -2/\sqrt{6}$; $(n_1)_2 = -1/\sqrt{6}$; $(n_1)_3 = -1/\sqrt{6}$

Exercise 10:

The equation of the plane $ax_1 + bx_2 + cx_3 = d$ is given by

$$3x_1 + 6x_2 + 2x_3 = 12 \quad (F(x_i) = 3x_1 + 6x_2 + 2x_3 - 12)^*$$

and the unit normal vector by the normalized gradient of this function,

$$\boldsymbol{n} = \frac{\nabla F}{\|\nabla F\|} = \frac{3}{7}\boldsymbol{e}_1 + \frac{6}{7}\boldsymbol{e}_2 + \frac{2}{7}\boldsymbol{e}_3$$

Thus, the stress vector is determined by the fo0llowing multiplication

$$\begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 7 & -5 & 0 \\ -5 & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} 3 \\ 6 \\ 2 \end{pmatrix} = \frac{1}{7}\begin{pmatrix} -9 \\ 5 \\ 10 \end{pmatrix} \quad \Rightarrow \boldsymbol{t} = -\frac{9}{7}\boldsymbol{e}_1 + \frac{5}{7}\boldsymbol{e}_2 + \frac{2}{7}\boldsymbol{e}_3$$

*Introduce in $ax_1 + bx_2 + cx_3 = d$ the values of the coordinates (one at a time) and express the coefficients in terms of d.

Problem 1: It can be solved in two ways

1. According to the given stress tensor, the stresses $\sigma_{13} = \sigma_{23} = 0$.
   Thus, one of the principal stresses is *C*. The other ones can be obtained using:

$$\sigma_{1,3} = \frac{\sigma_{11} + \sigma_{22}}{2} + \sqrt{(\frac{\sigma_{11} - \sigma_{22}}{2})^2 + \sigma_{12}^2} = -2C \pm 3C\sqrt{2} \qquad \text{(a)}$$

Thus, $\sigma_1 = -2C + 3C\sqrt{2}; \quad \sigma_2 = C; \quad \sigma_3 = -2C - 3C\sqrt{2}$ (b)

The maximum shear stress is

$$\max(shear\ stress) = \frac{\sigma_1 - \sigma_3}{2} = \frac{\left(-2C + 3C\sqrt{2}\right) - \left(-2C - 3C\sqrt{2}\right)}{2} = 3C\sqrt{2} \qquad \text{(c)}$$

2. **Solve the eigenvalue problem**

$$[\sigma] = \begin{pmatrix} -5C & -3C & 0 \\ -3C & C & 0 \\ 0 & 0 & C \end{pmatrix}$$

$$I_1(\sigma) = \sigma_{11} + \sigma_{22} + \sigma_{33} + = -3C$$

$$I_2(\sigma) = \begin{vmatrix} -5C & -3C \\ -3C & C \end{vmatrix} + \begin{vmatrix} C & 0 \\ 0 & C \end{vmatrix} + \begin{vmatrix} -5C & 0 \\ 0 & C \end{vmatrix} = -5C^2 - 9C^2 + C^2 - 5C^2 = -18C^2$$

$$I_3(\sigma) = \begin{vmatrix} -5C & -3C & 0 \\ -3C & C & 0 \\ 0 & 0 & C \end{vmatrix} = -5C\begin{vmatrix} C & 0 \\ 0 & C \end{vmatrix} + 3C\begin{vmatrix} -3C & 0 \\ 0 & C \end{vmatrix} + 0\begin{vmatrix} -3C & 0 \\ C & 0 \end{vmatrix} = -5C^3 - 9C^3 = -14C^2$$

Characteristic equation

$$\lambda^3 + 3C\lambda^2 - 18C^2\lambda \ + 14C^3 = 0$$

By observing the constant component, C is one of its multipliers. Thus, we have one of the roots (principal stresses) equal to *C*. Next, we can divide the characteristic equation by $(\lambda - C)$ to obtain the following,

$$\lambda^3 + 3C\lambda^2 - 18C^2\lambda \ + 14C^3 = \left(\lambda - C\right)\left(\lambda^2 + 4C\lambda - 14C^2\right)$$

Equation $\lambda^2 + 4C\lambda - 14C^2 = 0$ has roots given by (a) and thus, max shear stress by (c).

Problem 2:

1. **We examine the three equations of equilibrium:**

$\sigma_{ij,j} + f_i = 0$

$$\frac{\partial\sigma_{11}}{\partial x_1} + \frac{\partial\sigma_{12}}{\partial x_2} + \frac{\partial\sigma_{13}}{\partial x_3} + f_1 = -4x_1 + 4x_1 + 0 + f_1 = 0 \quad \Rightarrow \quad f_1 = 0$$

$$\frac{\partial\sigma_{21}}{\partial x_1} + \frac{\partial\sigma_{22}}{\partial x_2} + \frac{\partial\sigma_{23}}{\partial x_3} + f_2 = 4x_2 - 4x_2 + 0 + f_2 = 0 \quad \Rightarrow \quad f_2 = 0$$

$$\frac{\partial\sigma_{31}}{\partial x_1} + \frac{\partial\sigma_{32}}{\partial x_2} + \frac{\partial\sigma_{33}}{\partial x_3} + f_3 = -3 + 0 + 3 + f_3 = 0 \quad \Rightarrow \quad f_3 = 0$$

Thus for the given stress field, the body forces should be zero.

2. **Stress values at** *P* (4, -4, 7)

$\sigma_{11} = -2(4)^2 + 3(-4)^2 - 5(7) = -19$

$\sigma_{22} = -2(-4)^2 = -32$

$\sigma_{33} = 3(4) - 4 + 3(7) - 5 = 24$

$\sigma_{12} = 7 + 4(4)(-4) - 7 = -64$

$\sigma_{13} = -3(4) - 4 + 1 = -15$

$\sigma_{23} = 0$

$$\Rightarrow \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & 0 \\ \sigma_{13} & 0 & \sigma_{33} \end{pmatrix} = \begin{pmatrix} -19 & -64 & -15 \\ -64 & -32 & 0 \\ -15 & 0 & 24 \end{pmatrix}$$

Normal vector at P:

Gradient of $F(\boldsymbol{x}) = x_1^2 + x_2^2 + x_3^2 - 81$

$$\boldsymbol{n} = \frac{\nabla F(\boldsymbol{x})}{\|\nabla F(\boldsymbol{x})\|}$$

$$\Rightarrow \quad \boldsymbol{n} = \frac{(2x_1, 2x_2, 2x_3)^T}{\left[(2x_1)^2 + (2x_2)^2 + (2x_3)^2\right]} = \frac{1}{9}(4, -4, 7)^T$$

Stress vector:

$$\Rightarrow \begin{pmatrix} t_1 \\ t_2 \\ t_3 \end{pmatrix} = \begin{pmatrix} -19 & -64 & -15 \\ -64 & -32 & 0 \\ -15 & 0 & 24 \end{pmatrix} \frac{1}{9} \begin{pmatrix} 4 \\ -4 \\ 7 \end{pmatrix} = \frac{1}{9} \begin{pmatrix} (4)(-19) + (-4)(-64) + (7)(-15) \\ (4)(-64) + (-4)(-32) + (7)(0) \\ (4)(-15) + (-4)(0) + (7)(24) \end{pmatrix} = \frac{1}{9} \begin{pmatrix} 75 \\ -128 \\ 108 \end{pmatrix} = \begin{pmatrix} 8.33 \\ -14.23 \\ 12.00 \end{pmatrix}$$